Creating Learning Spaces

The Annual Clements/Foyster Lecture

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This paper addresses the conference theme of *Identities, cultures and learning spaces* by exploring how learning spaces might be theorised, how they are created, and how teachers and students negotiate their identities within these spaces. Examples of mathematics classroom practice and teacher learning and development are analysed using concepts derived from sociocultural theories of learning to consider what it might mean to speak of "technology enriched learning spaces". The analysis adapts Valsiner's zone theory to study interactions between teachers, students, technology, and the teaching-learning environment.

The conference theme of Identities, cultures and learning spaces reflects growing interest within the mathematics education research community in social and cultural aspects of learning. This "social turn" in the human sciences (Lerman, 2000; Sfard & Prusak, 2005) is marked by the emergence of theories that view learning as a form of participation in social and cultural practices rather than as an internal mental process. Over several years my colleagues and I have carried out a series of studies informed by sociocultural theories of learning to investigate the teacher's role in creating a classroom culture of mathematical inquiry (Goos, 2004), the dynamics of student discussion in small group problem solving (Goos, Galbraith, & Renshaw, 2002), the use of technology as a tool that mediates teaching and learning interactions (Goos, Galbraith, Renshaw, & Geiger, 2003), and the development of teachers' pedagogical identities (Goos, 2005a, 2005b). Each study, in its own way, invites us to think about the transformative possibilities of learning through interaction with other people and with the material and representational tools offered by the immediate learning environment and the culture that precedes us. It is in this sense, and through drawing on these studies, that I want to explore how learning spaces might be theorised, how they are created, and how individuals negotiate their identities within these spaces. To do this I will briefly sketch the historical development of key sociocultural concepts, explain how these concepts were elaborated or adapted for use in some of the studies mentioned above, and present analyses of several examples of classroom practice and teacher learning and development.

Sociocultural Theories of Learning

It is generally accepted that sociocultural theories trace their origins to the work of the Russian psychologist Vygotsky in the early 20th century (Forman, 2003). Wertsch (1985), one of the first scholars to interpret this work to Western researchers, identified three general themes at the heart of Vygotsky's theoretical approach. First of these is a reliance on a genetic or developmental method: in other words, to understand mental phenomena we need to concentrate on the process of growth and change rather than the product of development. The second theme is concerned with the social origins of higher mental functions: these appear first between people, on the social plane, and then within an individual, on the psychological plane. The third theme claims that mental processes are mediated by tools and signs, such as language, writing, systems for counting and

calculating, algebraic symbol systems, diagrams, and so on. In connection with these three themes Vygotsky analysed the concept of the Zone of Proximal Development (ZPD), a symbolic space where social phenomena are transformed into psychological phenomena. He proposed that the ZPD is created when a child's interaction with an adult or more capable peer awakens mental functions that have not yet matured and thus lie in the region between actual and potential developmental levels.

In the West, early attempts to apply Vygotsky's theory in educational research led to studies of how children learned through collaborative interaction with adults, and it became common to use the term "scaffolding" to describe the interaction between adult and child within the ZPD (e.g., Bruner, 1986; Rogoff & Wertsch, 1984). However, this first generation of Vygotskian inspired research was limited in that it relied on literal notions of internalisation of the interchange between child and adult, rather than more subtle semiotic mechanisms that might account for the transformation and appropriation of meanings during these interchanges. Through the 1980s and 1990s, more sophisticated interpretations began to emerge in a second generation of research that extended and enriched the emerging sociocultural framework by giving attention to the institutional context of social interactions, the importance of thinking are closely linked to forms of social practice (Forman, Minick, & Stone, 1993).

In addition to these developments, contemporary sociocultural theory acknowledges that learning involves increasing participation in a community of practice composed of experts and novices (Lave & Wenger, 1991). The concept of a community of practice was not originally focused on school classrooms, nor on pedagogy. However, Lave and Wenger's concern for "the whole person acting in the world" (p. 49), and their emphasis on changing participation and identity transformation, resonates with elements of the emergent social turn in educational research referred to above. In later work, Wenger (1998) elaborated on the concepts of identity and community as the foundations of a social theory of learning, and his ideas have been taken up by mathematics education researchers to analyse the identities developed by students in relation to particular classroom communities (Boaler, 2000), and by teachers in relation to professional development programs that promote innovative practices (Gómez, 2002; Graven, 2004). Some have claimed that the notion of identity is problematic without an operational definition that goes beyond timeless, essentialist references to "what kind of person" an individual is (see Sfard & Prusak, 2005). Yet Wenger's description of identity as "a way of talking about how learning changes who we are" (p. 5) does seem to capture the dynamic nature of identity as something that is constantly negotiated through the interplay of one's lived experience in the world and how we and others discursively interpret that experience.

Technology Enriched Learning Spaces

Research on technology enriched mathematics learning has grown in significance in recent years, especially since the advent of handheld technologies such as graphics calculators and their peripherals (Arnold, 2004). This coming of age is reflected in the inclusion, for the first time, of two separate chapters on technology in mathematics education in the most recent MERGA review of research in Australasia (Forster, Flynn, Frid, & Sparrow, 2004; Goos & Cretchley, 2004), and the commissioning of the 17th ICMI Study on "Digital Technologies and Mathematics Teaching and Learning". However,

despite the obvious research interest in this area, few studies have investigated how students use technology to learn mathematics in specific classroom contexts (see Doerr & Zangor, 2000 and Guin & Trouche, 1999, for exceptions) and how their teachers learn to integrate technology into their classroom practice (Windschitl & Sahl, 2002). What might it mean to speak of "technology enriched learning spaces", for students and teachers, in terms of the sociocultural perspective outlined in the previous section? To address this question, let me refer to a series of studies carried out with several colleagues that involved teachers and students in Australian secondary school mathematics classrooms (see Galbraith & Goos, 2003; Goos, 2005a, 2005b; Goos et al., 2003).

In our research program we adapted Valsiner's (1997) zone theory, originally designed to conceptualise child development, to apply to interactions between teachers, students, technology, and the teaching-learning environment. This framework re-interprets and extends Vygotsky's concept of the Zone of Proximal Development to incorporate the social setting and the goals and actions of participants. Valsiner regards the ZPD as a set of possibilities for development that are in the process of becoming actualised as individuals negotiate their relationship with the learning environment and the people in it. To explain how the actual emerges from the possible, Valsiner proposes two additional zones, the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA comprises activities, objects, or areas in the environment in respect of which the person's actions are promoted. For learning to occur, the ZPA must be consistent with the individual's possibilities for development (ZPD) and must promote actions that are feasible within a given ZFM. Thus we can think of the overlap between the three zones as a *learning space* — noting that the zones themselves are not physical spaces, but abstractions that are constantly "filled in" with new people, actions, places and meanings.

In the case of *student* learning the ZFM symbolises what the teacher allows, and is fashioned by environmental constraints such as the particular learning experiences provided and the types of resources these incorporate, and the "rules" by which the classroom is run. The ZPA represents the actions that the teacher is specifically promoting in the learner to bring about development of new skills. When we consider how *teachers* learn and develop, the ZFM can be interpreted as constraints within the school environment such as students (their behaviour, motivation, perceived abilities), access to resources and teaching materials, and curriculum and assessment requirements, while the ZPA represents teaching approaches that might be promoted by pre-service teacher education, professional development activities, and colleagues in the school setting.

Our research also investigated the tool mediated nature of learning in technology enriched classrooms. From a sociocultural perspective, technologies such as computers and graphics calculators are cultural tools that not only re-organise the way that students think but also transform classroom social practices (Berger, 1998). We developed the metaphors of technology as *master*, *servant*, *partner*, and *extension of self* to describe various modes of working with technology. Teachers and students can see technology as *a master* if their knowledge and competence are limited to a narrow range of operations. In the case of students, subservience may become dependence if lack of mathematical understanding prevents them from evaluating the accuracy of the output generated by the calculator or computer. For teachers, lack of knowledge and experience can make them reluctant to allow students to use technology to explore new mathematical concepts. Technology is a *servant* if used by students or teachers only as a fast, reliable replacement for pen and paper calculations. However, when teachers develop an affinity for technology as a *partner*, there is potential for increasing the power that students exercise over their learning by providing access to new kinds of tasks or new ways of approaching existing tasks. For students, this cognitive re-organisation effect may involve using technology to facilitate understanding, explore different perspectives, or mediate mathematical discussion in the classroom. Technology becomes an *extension of self* when seamlessly incorporated into the user's pedagogical or mathematical repertoire, such as through the integration of a variety of technology resources into the everyday practices of the mathematics classroom. These metaphors can be used to investigate some of the features of *technology enriched learning spaces* in terms of qualitative differences in the nature of students' learning experiences and in the pedagogical approaches taken by their teachers.

Creating Learning Spaces for Students

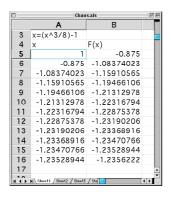
The examples presented in this section come from a study that investigated how teachers used technology in mathematics classrooms and the implications this has for students' learning. Episodes are taken from the Year 11 classroom of one teacher (Steve) who was an expert and innovative user of technology.

Engagement

The first episode spans two consecutive lessons in which students were introduced to iteration as one of the central ideas of chaos theory. This topic was presented as a teacherprepared booklet containing a series of spreadsheet examples and tasks for students to work through at their own pace. One task involved using iterative methods to find approximate roots of equations such as $x^3 - 8x - 8 = 0$. The equation may be expressed in the form x = F(x), and a first approximation to the solution is obtained by estimating the point of intersection of the curves y = x and y = F(x). This approximate solution is used as the initial value in a two column spreadsheet, where the first column provides input *x*-values for F(x) in the second column, and the output of F(x) becomes the input of subsequent iterations. Figure 1 shows the calculation when $F(x) = \frac{x^3}{8} - 1$. Depending on the way in which the original equation is re-arranged and the initial value chosen, the iteration may converge on a solution (as in Figure 1), or generate increasingly divergent outputs and hence no solution (as in Figure 2).

In the first lesson I observed a group of four students clustered around a laptop computer, sharing the responsibilities of pencil-and-paper and keyboard work. Instead of following the written instructions on how to use the spreadsheet method, they launched the graphing software installed on the laptop computer and plotted $y = \frac{x^3}{8} - 1$ on the same axes as y = x. Although three intersection points were clearly visible (see Figure 3), they zoomed in on only one of these to find the *x*-coordinate and obtained an approximate value of 3.24. Ignoring the other solutions, they used their graphics calculator's Equation Solver with x = 3.24 entered as an initial guess. The group accepted this as "the" solution — there was no attempt to explore other two intersections. The lesson ended before this anomaly

could be explored further. In this segment, the students deferred to the graphics calculator (technology as *master*) and blindly accepted the output produced by the Equation Solver without monitoring its reasonableness in the light of the graphical evidence before them.



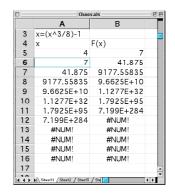


Figure 1. Spreadsheet method for equation solving with initial value x = 1: converges on a solution.

Figure 2. Spreadsheet method for equation solving with initial value x = 4: no solution.

At the start of the next lesson I mentioned to Steve that this group of students had not used spreadsheets at all. He then repeated the task instructions to the whole class, emphasising the importance of the spreadsheet approach. This moved the students away from their uncritical acceptance of the Equation Solver answer from the previous lesson towards using technology as a *servant* in order to demonstrate the utility of a spreadsheet in performing time consuming calculations.

The students started on the cubic problem again, this time using a spreadsheet. However, their answer, -1.24 (see Figure 1), did not correspond to the graphical result obtained earlier. They tried scrolling down the spreadsheet in the hope that they would find the other solutions, but when this was not successful they asked the teacher to explain how the spreadsheet worked. Steve did so, and challenged the students to use the spreadsheet approach to find all three solutions. By juxtaposing the spreadsheet, showing only one solution, with the graph, which displayed all three, the teacher attempted to have the students use technology as a *partner* to explore different ways of solving cubic equations and engage with the task in the way he had originally intended.

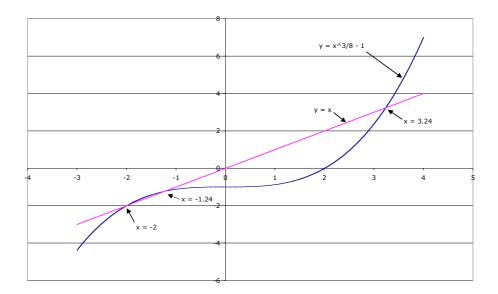


Figure 3. Graphical representation of iterative solution to $x^3 - 8x - 8 = 0$.

The students found that trying different initial values made no difference: the iteration either converged on -1.24 or became increasingly large. Again, they asked the teacher for help. Because Steve was aware that other groups had rearranged the cubic equation in different ways and thus obtained different solutions, he suggested the four students take a walk around the class to see how others were tackling the task. His intervention reinforced the role of technology as a *partner* in mediating mathematical discussion between students.

The students dispersed to consult with other groups, and discovered two other ways of rearranging the equation, $x = \sqrt[3]{8x+8}$ and $x = \frac{8x+8}{x^2}$. These gave the "missing" spreadsheet solutions of 3.24 and -2 respectively. Piecing together the information they had obtained, the group set up the relevant spreadsheets and confirmed they had found all three solutions. This resulted in some excitement as no other group had managed to do so.

Making a spur of the moment decision, Steve asked the group to connect their laptop computer to the data projector and present their findings to the class. Although they had no time to prepare explanations, a communally constructed argument emerged through questioning by the teacher and other students. Mathematical and communications technologies were thus seamlessly integrated to support argumentation, suggesting that technology became an *extension of self* for this group of students.

In Steve's classroom students were free to collaborate with peers on problems for which technology could assist with the mathematical processing required. A more public forum for interaction occurred when one group of students presented their solution to the whole class. Thus the teacher offered a Zone of Promoted Action to engage with students' ZPDs by enabling them to propose and defend mathematical conjectures, a form of mathematical activity highly valued in this classroom. The ZPA was also consistent with the Zone of Free Movement representing the classroom learning environment in the sense that there were no restrictions on the technology resources available, nor on students' access to the knowledge resources of their peers.

Resistance

Boaler (2000) claims that "learning to be a successful student of mathematics involves learning the rules of the school mathematics game and forming a learning identity that fits with the norms of the classroom community" (p. 394). The second episode demonstrates that students are not always willing to learn the rules, which in this classroom involved participating in mathematical discussion and using technology in the ways promoted by the teacher. George was a student who had consistently rejected all attempts to have him discuss his thinking with peers, and he seemed content to adopt a passive role and wait for the teacher to hand over the required knowledge. George also displayed no interest in working with any kind of technology, until a lesson when students were asked to program their calculators to find the angle between two vectors. Several volunteers then demonstrated their programs via the overhead projection unit so that the class could compare and evaluate these different approaches (see examples in Figure 4).



Figure 4. Different student programs for finding the angle between two vectors (initial screens only).

George had produced the program (which he named "Dodge") that begins with the second screen in Figure 4. After the vectors are entered, the user is presented with the first two screens in Figure 5. The correct answer is only displayed if option 1 is selected in response to the question shown on screen 3 of Figure 5; otherwise, the program "refuses" to provide the answer (screen 4 of Figure 5).

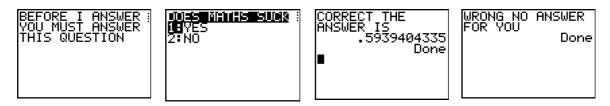


Figure 5. Presentation of the Dodge program.

Although George appeared to be displaying resistance to, rather than affiliation with, the values of the classroom, this was expressed in the very manner encouraged by the teacher — by using technology as a means of communicating mathematical ideas to peers in a whole class forum. Initially the teacher designed ZPA did not overlap with George's ZPD: he rejected the teacher's invitation to mathematical inquiry and the learning identity that went with it. However, the admiring response of his peers seemed to persuade this student in subsequent lessons to become engaged in other technology mediated classroom discussions, some of which included his presentation of more sophisticated versions of the Dodge program and other animated programs he had created. In Wenger's (1998) terms, it seemed that George layered these experiences and their social interpretation by others (peers and teacher) to negotiate a new learning identity as a "successful" student.

Creating Learning Spaces for Teachers

Understanding how teachers learn from their experiences in different environments is a complex business because teachers' own schooling, their university pre-service program and practicum sessions, and initial professional socialisation can produce conflicting images of mathematics teaching. Novice teachers often resolve these conflicts by abandoning the innovative approaches they may have learned about during their pre-service program when they enter the more conservative setting of the school. Valsiner's zone theory provides a way of analysing how these influences on teacher development come together to shape novice teachers' pedagogical identities. This is illustrated by an example from a study that investigated how teachers who had graduated from a technology rich pre-service program integrated computers and graphics calculators into their mathematics teaching practice. The episodes below capture snapshots of one participant's experience during the practicum and toward the end of his first year of full-time teaching.

Pre-Service Teaching

The school where Adam completed his practice teaching sessions had recently received significant government funding to refurbish classrooms in the Mathematics Department and buy resources such as graphics calculators, data logging equipment, and software. Every mathematics classroom was equipped with computers connected to the Internet, a data projector, and a TV monitor for projecting graphics calculator screen output. A hire scheme provided calculators to all students in the final two years of secondary school, and there were also sufficient class sets of calculators for use by younger classes. Some of these changes had been made in response to new mathematics syllabuses that mandated the use of computers or graphics calculators in teaching and assessment programs. Thus the school and curriculum environment offered a Zone of Free Movement that seemed to afford the integration of technology into mathematics teaching.

Adam had previously worked as a software designer and was a confident user of computers and the Internet. Although he had not used a graphics calculator before starting the pre-service course, he quickly became familiar with its capabilities and with the encouragement of his Supervising Teacher began to incorporate this and other technologies into his mathematics lessons. In theoretical terms, then, the Zone of Promoted Action organised by the Supervising Teacher was consistent with that offered by the university course and also with the ZPD that defined Adam's possibilities for development. Nevertheless, Adam wondered how he could prevent students from becoming dependent on the technology, as they might simply "punch it into their calculator and get an answer straight away". At this stage he had only ever used technology in his teaching, or observed its use by other teachers, as a tool for saving time in plotting graphs and performing complicated calculations, or for checking work done first by hand — that is, as a *servant*.

Beginning Teaching

After graduation Adam was employed by the same school where he had completed his practicum. One might expect that Adam experienced a seamless transition from pre-service to beginning teacher; yet this was not the case. As Adam was now teaching a full timetable he came to realise that not all classes could be accommodated in the well-equipped Mathematics Department building, and lessons he taught in other classrooms could not

incorporate computers or the Internet. He also discovered that inadequate infrastructure and lack of technical assistance compromised the reliability of the school's computer network and hardware.

Despite these difficulties Adam continued to integrate technology into his teaching wherever possible. The following example comes from a Year 11 lesson in which he planned to teach about the effects of the constants a, b, and c on the graph of the absolute value function y = a|x + b| + c. Although he clearly had specific goals in mind, the lesson was driven by the students' questions and conjectures rather than a predetermined stepby-step plan. The students first predicted what the graph of y = |x| would look like, based on their knowledge of the y = x function and the meaning of absolute value, and then compared their prediction with the graph produced by their graphics calculators. One student noticed that the graph involved a reflection in the y-axis and she asked how to "mirror" this effect in the x-axis. Immediately another student suggested graphing y = -|x|, and the teacher followed this lead by encouraging the class to investigate the shape of the graph of y = a|x| and propose a general statement about their findings. The lesson continued with students making similar suggestions for examining the effects of b and c.

After the lesson Adam explained that he had developed a much more flexible teaching approach:

I had a rough plan and we kind of went all over the place because we found different things, but I think that's better anyway. Because the kids are getting excited by it and they're using their calculators to help them learn.

Instead of viewing technology purely as a *servant*, Adam now maintained that the role of technology was "to help you [i.e., students] get smarter" by giving students access to different kinds of tasks that build mathematical understanding. His comments reflect a shift towards viewing technology as a *partner*.

These observations suggest that Adam's possibilities for development — his ZPD had expanded since the practicum, and that this potential would be promoted with the assistance of his colleagues (ZPA). But this was the case for only some colleagues. Adam discovered that many of the other mathematics teachers were unenthusiastic about using technology and favoured teaching approaches that he claimed were based on their faulty belief that learning is linear and teacher-directed rather than richly connected and studentled. Because he disagreed with this approach, Adam deliberately ignored the worksheet provided for the lesson by the teacher who coordinated this subject, which represents a mode of working with technology as a *servant*. The worksheet led students through a sequence of exercises where they were to construct tables of values, plot graphs by hand, and answer questions about the effects of each constant in turn. Only then was it suggested that students might use their graphics calculators to check their work. Conflicting pedagogical beliefs were a source of friction in the staffroom, and this was often played out in arguments where the teacher in question accused Adam of not teaching in the "right" way. Compared with his earlier experience as a pre-service teacher, Adam now found himself in a more complex situation that required him to defend his instructional decisions while negotiating a harmonious relationship with several colleagues who did not share his beliefs about learning. Adam explained:

[[]Now I'm willing] to stand up and say "This is how I am comfortable teaching". I just walk away now because we've had it over and over and the kids are responding to the way I'm teaching them. So I'm going to keep going that way.

In terms of Valsiner's zone framework, Adam became aware of conflicts between his ZFM, which sometimes restricted his access to technology, a ZPA that promoted, at best, fairly mundane uses of technology in his teaching, and his personal ZPD. He responded by working within the limits of the ZFM while paying attention only to those aspects of the Mathematics Department's ZPA that were consistent with his own beliefs and goals (his ZPD) and also with the technology-focused ZPA offered by the university pre-service course. We could say that his professional identity developed to the extent that he was able to reconcile his pedagogical beliefs with the constraints of his teaching environment.

Discussion

The opportunities that teachers provide for student learning are affected by ways in which they interpret and analyse problems of practice. How do teachers justify and enact decisions about which strategies and resources to use in their classrooms? How do they reconcile potential contradictions between their own pedagogical knowledge and beliefs and those of their colleagues? How do they interpret aspects of their environments that support or inhibit particular teaching approaches? These questions, when framed within a sociocultural perspective, allow us to systematically investigate student learning and teacher development through the application of Valsiner's zone theory — where the Zone of Proximal Development represents aspects of development that are moving from possibility to actuality, the Zone of Free Movement environmental constraints, and the Zone of Promoted Action the nature of the specific activities designed to promote new skills and understanding. In the context of technology enriched teaching and learning, the metaphors of master, servant, partner and extension of self provide a language for describing how technology, as a cultural tool, enters into the mathematical practices of the classroom.

Different ZPD/ZFM/ZPA configurations produce different kinds of learning spaces for students, and for teachers. Consider first the implications for students' learning. While teachers construct activities (ZPAs) consistent with their learning goals for students, these planned activities may or may not be consistent with the learning resources accessible to students (ZFMs). For example, students may lack access to technology resources, the range of such resources may be restricted, or ways in which students are permitted to exploit the potential of available resources may be curtailed. This was the case in the classrooms of some of Adam's colleagues, who either did not use graphics calculators in their teaching at all or kept tight rein on its use (setting ZFM = ZPA). In contrast, Steve's pedagogical approach (ZPA) of encouraging students to propose conjectures, defend solutions, and suggest alternatives was consistent with a classroom ZFM that gave unrestricted access to various types of technology and permitted students to share material and intellectual resources. Even so, it was possible for students like George to resist the teacher's efforts and negotiate new learning identities on their own terms.

Teachers' learning can also be conceptualised in terms of changing ZPD/ZFM/ZPA relationships and this provides a useful way of analysing how teachers develop their pedagogical identities in response to innovative practices, such as those involving technology. For these teachers, the ZFM can be interpreted as their institutional context, the ZPA represents the efforts of other people to promote particular teaching approaches, and the ZPD is influenced by their knowledge of how to integrate technology into their teaching and their pedagogical beliefs. As with students, learning occurs in the overlap

between the zones, that is, when the ZPA is consistent with the individual's need and potential (ZPD) and when it promotes teaching actions believed to be feasible within the effective ZFM. The case study of Adam illuminated issues facing novice teachers who are knowledgeable and enthusiastic about using technology but encounter obstacles in their professional environment (ZFM), or in the example provided by more experienced colleagues (ZPA), that hinder implementation of preferred teaching approaches. Yet Adam remained an active agent in negotiating his own identity as a teacher, neither yielding to environmental constraints nor reproducing the practices of other teachers, but instead interpreting these conditions in the light of his own professional goals and beliefs.

MERGA: Creating a Learning Space for Researchers?

The examples I have used in this paper relate to technology enriched mathematics education, but Valsiner's zone theory can be applied to human development in general — including, perhaps, the development of mathematics education researchers. Each of us could probably describe a set of possibilities for our own development in the near future (ZPD), as well as structures within the academic environment that, once internalised, constrain our actions in culturally expected ways (ZFM). To what extent does MERGA offer opportunities to learn — a ZPA — that nurture individuals' aspirations within the broadest, most inclusive professional boundaries? Where do the actions promoted by MERGA sit within an external environment increasingly structured by accountability and quality measures? Most importantly, how does MERGA membership allow researchers to build academic identities through experiences such as attending conferences and presenting papers, and through reflecting on these experiences to understand how we and others see ourselves? These are questions that deserve our continuing attention if MERGA is to remain "a living context for learning" (Anthony, 2004, p. 11).

Acknowledgements. My thanks to Peter Galbraith, Vince Geiger and Peter Renshaw for sharing insights that helped shape this paper, and to all the teachers and students who have participated in our research program.

References

- Anthony, G. (2004). MERGA: A community of practice. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville, pp. 2-15). Sydney: MERGA.
- Arnold, S. (2004). Mathematics education for the third millennium: Visions of a future for handheld classroom technology. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville, pp. 16-28). Sydney: MERGA.
- Berger, M. (1998). Graphics calculators: An interpretive framework. For the Learning of Mathematics, 18(2), 13-20.
- Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior*, 18, 379-397.
- Bruner, J. (1986). Actual minds, possible worlds. Cambridge, MA: Harvard University Press.
- Doerr, H. M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics*, *41*, 143-163.
- Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), A research companion to principles and standards for school mathematics (pp. 333-352). Reston, VA: National Council of Teachers of Mathematics.
- Forman, E. A., Minick, N., & Stone, C. A. (1993). Contexts for learning: Sociocultural dynamics in children's development. New York: Oxford University Press.
- Forster, P. Flynn, P. Frid, S., & Sparrow, L. (2004). Calculators and computer algebra systems. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in mathematics education in Australasia 2000-*

2003 (pp. 313-336). Flaxton, QLD: PostPressed.

- Galbraith, P., & Goos, M. (2003). From description to analysis in technology aided teaching and learning: A contribution from zone theory. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia, Geelong, pp. 364-371). Sydney: MERGA.
- Gómez, C. (2002). The struggles of a community of mathematics teachers: Developing a community of practice. In A. Cockburn & E. Nardi (Eds.), Proceedings of the 26th annual conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 9-16). Norwich, UK: PME.
- Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35, 258-291.
- Goos, M. (2005a). A sociocultural analysis of the development of pre-service and beginning teachers' pedagogical identities as users of technology. *Journal of Mathematics Teacher Education*, *8*, 35-59.
- Goos, M. (2005b). A sociocultural analysis of learning to teach. In H. Chick & J. Vincent (Eds.), Proceedings of the 29th annual conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 49-56). Melbourne: PME.
- Goos, M., & Cretchley, P. (2004). Computers, multimedia, and the Internet in mathematics education. In
 B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in mathematics education in Australasia 2000-2003* (pp. 151-174). Flaxton, QLD: PostPressed.
- Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, 49, 193-223.
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003) Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior*, 22, 73-89.
- Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational Studies in Mathematics*, 57, 177-211.
- Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers in Mathematical Learning*, *3*, 195-227.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). Westport CT: Ablex Publishing.
- Rogoff, B., & Wertsch, J. (1984). Children's learning in the "zone of proximal development". San Francisco: Jossey-Bass.
- Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.
- Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development.* (2nd ed.) New York: John Wiley & Sons.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge, MA: Cambridge University Press.
- Wertsch, J. V. (1985). Vygotsky and the social formation of mind. Cambridge, MA: Harvard University Press.
- Windschitl, M., & Sahl, K. (2002). Tracing teachers' use of technology in a laptop computer school: The interplay of teacher beliefs, social dynamics, and institutional culture. *American Educational Research Journal*, 39, 165-205.